Module 1 – Lecture 2:
Spatial Models of Disease Spread &
Public Health Control Measures
“To suppress and control the epidemic, countries must isolate, test, treat and trace”

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Director, World Health Organization (WHO)
Outline

1) Extensions of the SIR Model: Additional States

2) Spatial models and network theory

3) Public policy measures for controlling disease spread
   → Test, trace, and isolate
Review: SIR model

SIR Model:

- Susceptible population ($S$)
  - ...are infected by infected people at rate $\beta$
- Infected population ($I$)
  - recover (or die) at a rate $\gamma$
- Recovered/Removed population ($R$)

\[
\beta \text{ (infected population)} \rightarrow \gamma \text{ (recovered or removed population)}
\]
SIR Model:

Susceptible population \((S)\)  \(\xrightarrow{\beta} \)  Infected population \((I)\)  \(\xrightarrow{\gamma} \)  Recovered/Removed population \((R)\)

...are infected by infected people at rate \(\beta\)

\(\beta (S) (I)\)

\(\gamma\) recover (or die) at a rate \(\gamma\)

SEIR Model: Add an “exposed” but latent period before becoming infectious

Susceptible population \((S)\)  \(\xrightarrow{\beta} \)  Exposed population \((E)\)  \(\xrightarrow{\gamma} \)  Recovered/Removed population \((R)\)

Exposed population \((E)\)  \(\xrightarrow{\beta} \)  Infected population \((I)\)  \(\xrightarrow{\gamma} \)  Recovered/Removed population \((R)\)

\(\beta (S) (E)\)

\(\gamma\)
Relevant Extensions: Additional States

**SIR Model:**

\[ \begin{align*}
S & \xrightarrow{\beta} I \\
I & \xrightarrow{\gamma} R
\end{align*} \]

- **Susceptible population** (S)
- **Infected population** (I)
- **Recovered/Removed population** (R)

**SEPIR Model:** *Also add a “Pre-symptomatic but infectious” period*

\[ \begin{align*}
S & \xrightarrow{\beta} E \\
E & \xrightarrow{\beta} P \xrightarrow{\beta} I \xrightarrow{\gamma} R
\end{align*} \]

- **Susceptible population** (S)
- **Exposed population** (E)
- **Pre-symptom infectious population** (P)
- **Infected population** (I)
- **Recovered/Removed population** (R)

Relevant Extensions: Additional States

SIR Model:

Susceptible population ($S$) \( \xrightarrow{\beta} \) Infected population ($I$) \( \xrightarrow{\gamma} \) Recovered/Removed population ($R$)

SEPIR Model: Also add a "Pre-symptomatic but infectious" period

Susceptible population ($S$) \( \xrightarrow{\beta} \) Exposed population ($E$) \( \xrightarrow{\beta} \) Pre-symptomatic infectious population ($P$) \( \xrightarrow{\gamma} \) Infected population ($I$) \( \xrightarrow{\gamma} \) Recovered/Removed population ($R$)
Relevant Extensions: Additional States

SIR Model:

- Susceptible population \((S)\)
- Infected population \((I)\)
- Recovered/Removed population \((R)\)

\[
\begin{align*}
&\text{Susceptible population} (S) \\
&\text{Infected population} (I) \\
&\text{Recovered/Removed population} (R)
\end{align*}
\]

SIR Model with Different Infectious States (e.g. symptomatic & asymptomatic):

- Susceptible population \((S)\)
- Symptomatic population \((I_1)\)
- Asymptomatic population \((I_2)\)
- Recovered/Removed population \((R)\)

\[
\begin{align*}
&\text{Susceptible population} (S) \\
&\text{Symptomatic population} (I_1) \\
&\text{Asymptomatic population} (I_2) \\
&\text{Recovered/Removed population} (R)
\end{align*}
\]

- Rate of going into infectious state 2 due to contacting someone in infectious state 1

\[
\begin{align*}
&\beta_{11}, \beta_{12} \\
&\beta_{21}, \beta_{22}
\end{align*}
\]
Major Limitation of Basic SIR Model: Single Variable for Entire Population

Implicit assumption 1: Everyone contacts everyone

Implicit assumption 2: No randomness: describes average behavior of large populations
Spatial ("Individual-Based") Models

1) Spatial models allow for individuals to contact different people:

- Everyone contacts everyone
- Different contacts for different people

2) Often in spatial models, transmission of infection is probabilistic (aka "stochastic"): At each moment of time, for every contacting pair, the simulator "rolls the dice" to determine if a transmission occurs
Key Concepts from Network Theory: Nodes, Edges, & Degrees

**Key concept 1:** Networks (aka ‘graphs’) are made up of nodes connected by edges

- **Node:** represents a unit (e.g. a person)
- **Edge:** represents connections between nodes (e.g. contacts between people)
- **Degree** (of a node): the number of edges it connects to
- **Color** of a node can represent different states (e.g. susceptible or infectious)

*network from epidemix:*
Structure of a network can be characterized by its “degree distribution”

Randomly connected network

Clustered network

Hubs: Nodes with degree much greater than average

Average = 2 connections/node
**Question**: Which type of network do you think has faster disease spread?

**Answer**: Networks with more clustering (hub-like nodes) tend to spread disease faster, because disease spreads rapidly across the network after reaching a hub.
The “Mail Carrier” Network

Randomly connected network
(Shown for p=1: each contact leads to infection)

“Mail carrier” network
(Shown for p=1: each contact leads to infection)

$t = 3$

random network $\rightarrow$ spread is relatively slow
The Mail Carrier Network

Randomly connected network
(Shown for p=1: each contact leads to infection)

initial infected person

t = 3

random network \(\rightarrow\) spread is relatively slow

“Mail carrier” network
(Shown for p=1: each contact leads to infection)

initial infected person

t = 2

hub-like network: faster spread
Part 2: Public Policy Measures for Reducing Disease Spread
Example Case: China

Key question: What happened on these dates?

Reff = 3.88

Reff = 1.25

Reff = 0.32

data & fits from Wang et al., medRxiv, 2020
Control Measures of Progressive Strength

Social distancing/Shelter-at-home (imperfect):
Most people at home, but go to grocery store, imperfect compliance
Control Measures of Progressive Strength

Household quarantine:
If someone sick, whole household is quarantined at their home

- infected person
- contacts of infected person ("1st degree contacts")
- contacts of infected person’s contacts ("2nd degree contacts")
Centralized quarantine:
If someone sick, they & their contacts are quarantined at separate facilities

- contacts of infected person ("1st degree contacts")
- contacts of infected person’s contacts ("2nd degree contacts")
Test, Trace, Isolate/Quarantine

**Test:** Identify (by symptoms or invasive test) people with coronavirus

**Contact trace:** Trace (manually or digitally) contacts of infected people

**Quarantine:** Quarantine both infected people and their contacts

Can this work in practice?
Example Case: China

Key question: What happened on these dates?

Data & fits from Wang et al., medRxiv, 2020
Example Case: China

Social distancing imposed

$R_{eff} = 1.25$

Projection if had not done further measures

data & fits from Wang et al., medRxiv, 2020
Example Case: China

Data & fits from Wang et al., medRxiv, 2020

Centralized quarantine imposed

\[ R_{\text{eff}} = 3.88 \]  
\[ R_{\text{eff}} = 1.25 \]  
\[ R_{\text{eff}} = 0.32 \]
Will Test, Trace, Isolate/Quarantine Work in the U.S.?

Key Issues:

• Do we have enough employees to do manual identifications of cases and contacts?

• Are manual identifications fast enough?

→ Ideally want to identify infectious people immediately, before they infect too many others
Here’s How Apple and Google Plan to Track the Coronavirus Through Your Phone

The idea requires widespread adoption and broad testing of potentially infected people, whether government and public-health officials will get behind it.

Apple and Google Team Up to ‘Contact Trace’ the Coronavirus

The technology giants said they would embed a feature in iPhones and Android devices to enable users to track infected people they’d come close to.
1) Contact tracing (always happening while phone on):

Subject A has COVID-19 infection. No symptoms

Day 1

Home
Train
Work
Home

Time

A
B
C
D
E
F
G
H
I

Close Nearby

Ferretti et al, *Science*, 2020
Proposal for Digital Tracing

Ferretti et al, *Science*, 2020

2) Symptoms → request test

3) Test & isolate close contacts
Summary: Disease Modeling ...and Control Policy

1) Control measures attempt to suppress the outbreak: bring $R_{\text{eff}} < 1$

2) SIR models can be used to model disease outbreak ...and guide policy by simulating the effects of different control measures

3) Hub people or locations can increase the speed of spread ...but may also provide key targets for control policies (see lab)

4) Test, trace, & isolate: may be able to control the epidemic, especially if done digitally ...but models suggest it must be widely adopted, and privacy issues remain a challenge
A Few Places to Keep Informed

• *NY Times* coronavirus page:

• *Science and Nature*

• Johns Hopkins Coronavirus Resource Center:
  https://coronavirus.jhu.edu/

• Institute for Health Metrics and Evaluation (IHME)
  http://www.healthdata.org/
How to Learn More About Modeling (for juniors or your friends...)

• **BIS 20Q: Models in Biology**  
  A ‘greatest hits album’ of key models driving biology research

  Provides the key math needed to understand these studies

• **BIS 23A/B: Genome Hunters**  
  Course-based research class: microbial growth & genomics experiments, with bioinformatic analysis of resulting data

And coming soon...
An Interdisciplinary Quantitative Biology Major

Expected launch date: Fall 2021